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Highly accurate laser wavelength meter based on Doppler effect

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Abstract

We have built an accurate wavelength meter based on a Michelson interferometer characterized by a high stability velocity moving system. The unknown wavelength is determined from the Doppler frequency shifts of the output beams of the Michelson interferometer. The reference laser is a frequency stabilized helium–neon laser. A counting resolution of 2.6×10^{-9} for an integration time of 30 s has been obtained. The apparatus has been used to determine the wavelength of a second frequency stabilized helium–neon laser and the result has been compared to those given by two different methods: frequency beating in regards to the national reference and using a commercially available scanning-Michelson wavemeter. Taking into account the statistical errors, we achieved a relative accuracy on the unknown wavelength of 6.4×10^{-8} at 1σ.
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1. Introduction

For some applications involving laser sources, it is important to know their absolute wavelength accurately. In many cases, this is achieved by locking the laser to a known atomic or molecular transition in a vapor cell. This may not always be possible, especially when working with short-lived radioactive species or trapped ions. One solution is to measure the wavelength of the laser precisely by beating its frequency against that of a reference laser. Another way consists in feeding simultaneously the unknown probe wavelength λ_U and a well-known wavelength λ_R as reference into a two-beams interferometer with continuously varying optical path difference [1–3]. The wavelength of the probe laser can be computed with high accuracy by careful examination of the resulting interference fringes. The analysis boils down to the comparison of two sinusoidal fringe patterns. Obviously, the achievable accuracy scales linearly with the number of interference fringes. Simply counting the fringes gives the ratio $R = \lambda_U/\lambda_R$ of the wavelengths with an error of $\Delta R = 1/N$, where N denotes the number of fringes of the probe signal. Practical considerations limit the size of the device, and thus the number of fringes. The limited number of fringes counted during the measurement determines the ultimate accuracy for the device. In the most basic design [4], a displacement of the moving mirror of 15 cm gives a counting resolution equal to 10^{-6} (for an integration time τ of about 30 s). Compared to the other sources of errors, the counting resolution appears far outweigh all others. One way for improving the counting resolution (the conventional method) consists in including fractions of a fringe in the analysis at the beginning and at the end of the travel [5]. The resolution and accuracy are then limited by the common sources of errors in interferometry like the wavefront curvature, the nonlinearity of the phasemeter or the thermal drifts of the mechanics [6]. The originality of the present work lies in the fact that the value of λ_U is not linked to the ratio of the number of fringes but to the ratio of the fringe rates, i.e., the Doppler frequency shifts of the beams (reference and unknown lasers) coming from the Michelson interferometer. Hence, there is no need of fringe

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interpolations leading to a method far less sensitive to the errors cited below. On the other hand, velocity instabilities will induce phase noise onto the Doppler signal. For this purpose, we demonstrate a particular method for controlling the stability of the velocity of a translation stage which allows us to reach a counting resolution equal to 2.6×10^{-9} ($\tau = 30$ s). The method has been set up. A noise analysis and an evaluation of the statistical errors have been performed. We determined the wavelength of an unknown laser with an accuracy of 6.4×10^{-8} (1σ) leading to an improvement of two orders of magnitude compared to commercially available apparatus.

2. Principle, experimental setup and results

Consider a simple Michelson interferometer composed of two mirrors and a beamsplitter (Fig. 3). As the target mirror moves, the number of wavefronts emitted by the laser and reaching the detectors at the output of the interferometer within a certain time interval changes, resulting in a shift of the frequency of the electromagnetic wave ($\Delta\nu_R$). This shift is given by

$$\Delta\nu_R = \frac{2n_R V}{\lambda_R}, \quad (1)$$

where V is the velocity of the moving mirror and n_R the refractive index of the medium for a laser with a wavelength in vacuum equal to λ_R. We suppose that the laser has been calibrated and its wavelength in vacuum is known. Similarly, for a different laser with an unknown wavelength λ_u, $\Delta\nu_U = 2n_U V/\lambda_u$. As both lasers propagate through the interferometer simultaneously, the velocity of the mirror is the same for each beams. The value of the unknown wavelength is given by

$$\lambda_u = \frac{\Delta\nu_R}{\Delta\nu_U}\lambda_R\left(\frac{n_R}{n_U}\right). \quad (2)$$

Hence, the principle of our wavemeter is based on the measurement of the ratio of the Doppler frequency shifts induced on the laser beams coming from a reference laser and from an unknown laser.

Although the value of the unknown wavelength is independent of the value of the velocity (Eq. (2)), this is not the case of the noise due to the velocities instabilities, to photodetectors, the mechanical vibrations, the electronic boards and frequency laser fluctuations. This will result in instabilities onto the Doppler frequency shifts and hence onto the measurement of the laser wavelength. According to Eqs. (2) and (1), in order to reach a relative stability on λ of 10^{-8}, one need $\sigma_V/V = 10^{-9}$. Due to mechanical disturbances, it is impossible to reach a relative stability of the velocity for the moving target below to $10^{-4}\,\tau^{-1/2}$ over centimeter dynamic range in open loop. In close loop, the better translation stage available commercially can perform a velocity stability at $10^{-5}\,\tau^{-1/2}$ level. We develop a specific translation system which is capable to move with a velocity stability of $10^{-8}\,\tau^{-1/2}$ [7]. This system has been originally developed for the french moving-coil watt balance in order to redefine the unit of mass of the SI [8]. The system is a two-levels translation stage. The first level is a high precision translation stage from Aerotech (ALS20010) monitored by a magnetic linear motor. It is designed to have a velocity control at the 10^{-5} accuracy level. The drive system is composed of a linear brushless servo motor. The total travel is about 100 mm and the maximum velocity is 2 m s^{-1}. The second level is a piezoelectric translator (PZT). Its maximum travel range is 3 μm for an applied voltage of 40 V. A heterodyne laser source [9] emits two orthogonally polarized beams separated in frequency by $\delta\nu = |\nu_2 - \nu_1| = 20$ MHz (Fig. 1). The optical beams pass through a Michelson's interferometer. They are separated by a polarizing beamsplitter, recombined at the output of the interferometer and mixed by a mixing polarizer resulting in a signal s_3. This signal contains the information about the velocity. The ALS20010 has its own closed servo control system. The velocity control of the piezoelectric actuator is based on the use of a high frequency phase-shifting electronic device. It generates two synchronized signals s_1 and s_2 both at a frequency $\delta\nu$. It allows also to perform phase shifts on these signals. Signal s_2 is sent to a mixer to be phase-compared with s_3. Signal s_1 is used to synchronize a Bragg cell placed in the laser head which performs the two optical beams separated in frequency by $\delta\nu$ and orthogonally polarized thanks to a birefringent plate. A motion of the mirror with a velocity $V \pm \delta V_1$ is generated via the ALS20010. This motion leads to a Doppler phase shift per unit of time of the signal s_3 equal to $\Delta\phi_D \pm \delta\phi_1$, where $\delta\phi_1$ is the phase noise due to δV_1. Simultaneously phase shifts equal to $\Delta\phi_D \pm \delta\phi_2$ are made onto s_2 leading also to a velocity $V \pm \delta V_2$ for the movable mirror, but with an uncertainty $\delta V_2 \ll \delta V_1$. As $\delta\phi_2 \ll \delta\phi_1$, signal error at the output of the mixer is given by $\pm K \times \delta\phi_1$, where K is a constant factor. Result is that the stability of the velocity of the mirror is as fine as permitted by the

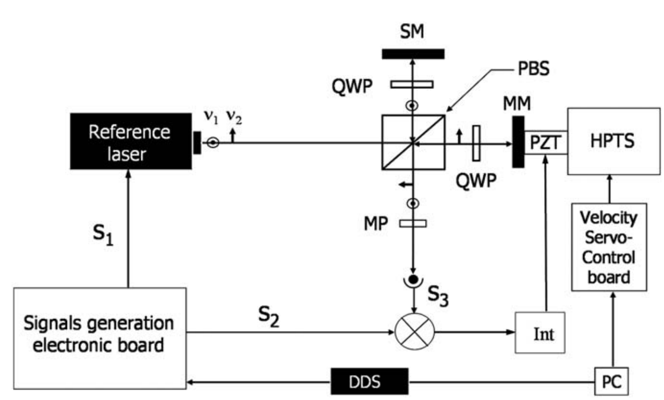

Fig. 1. Setup of the velocity servo control loop of the target mirror. SM: Stationary mirror, MM: movable mirror, PBS: polarization beamsplitter, QWP: quater wave plate, HPTS: high precision translation stage, PZT: piezoelectric actuator, Int: integrator, MP: mixing polarizer, DDS: digital direct signal generator. The signal generator is controlled by a computer (PC) via an IEEE communication port.

PZT actuator over the entire travel range of the ALS20010. Notice that it is possible to control the direction of displacement of the mirror by making phase shifts either on the signal s_2 either on the signal s_1. The limit of this method is due to the fact that it is not possible to equalize experimentally the two velocities perfectly and hence the total travel of the movable mirror is limited by the maximum travel range of the piezoelectric actuator. However, with a 3 μm travel range piezoelectric actuator and our electronic device, the movable mirror has been displaced over 80 mm without breaking the loop lock.

Characterization of the instabilities due to the different sources of noise can be expressed in the time domain as a function of the averaging time τ using the Allan standard deviation [10]. The signal coming from one part of the beam at the output of the Michelson's interferometer at a frequency $\delta v \pm \Delta v_D$ (where $\delta v = 20$ MHz and Δv_D is the Doppler frequency shift) is mixed with a reference signal at a frequency δv coming from our high frequency electronic circuit. The resulting signal is sent onto a high stability frequency counter HP53132A (Helwett–Packard) with a relative accuracy of 10^{-10}. The mirror is moved over 80 mm with a velocity of 2 mm s^{-1} leading to an acquisition time of 40 s. During this time, the Doppler frequency shift is measured over 1 s giving 1 measurement point. This step is repeated several times leading to about 1575 points of 1 s. Due to the calculus of the overlapped Allan variance $\sigma_y^2(\tau)$ (Eq. (3)), the maximum estimation time is about 400 s.

$$\sigma_y(\tau) = \left\{ \sum_{i=1}^{K-1} (y_{i+1}(\tau) - y_i(\tau))^2 \Big/ (2K-2) \right\}^{1/2}, \qquad (3)$$

where K is the number of samples of the Doppler shift measured using the frequency counter and $\overline{y_i}$ the normalized frequency differences of the K samples. Fig. 2 represents the Allan standard deviation of the Doppler shift corresponding to a velocity of 2 mm s^{-1}. We have got $K = 1575$ with a periodicity of 1 s. The standard deviation

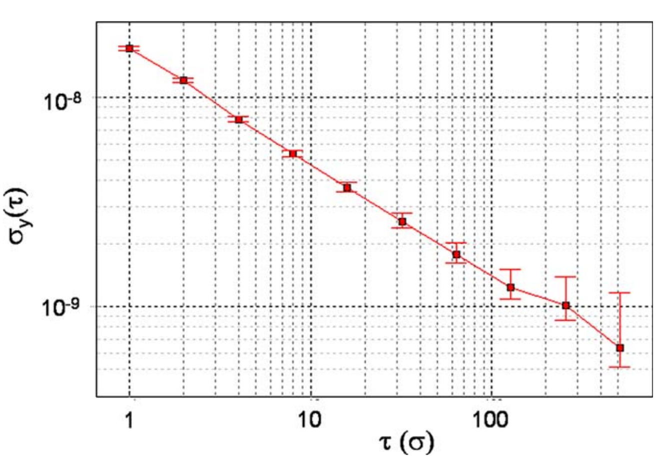

Fig. 2. Allan standard deviation for the Doppler shift of the reference laser.

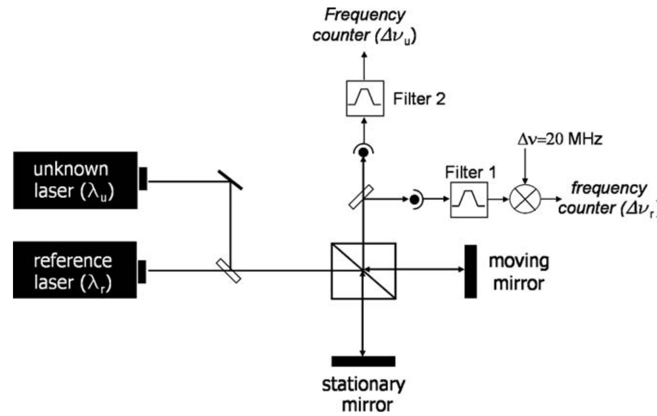

Fig. 3. Experimental setup of the wavemeter. Two frequency counters measure the Doppler frequency shifts due to the moving mirror for both lasers. The filter 1 is a band-pass filter centered around 20 MHz. The filter 2 is a low-pass filter (cut-off frequency = 30 kHz). As the reference laser has two optical components separated in frequency by 20 MHz, it is first demodulated before reaching the frequency counter. The unknown wavelength is determined thanks to Eq. (3).

$\sigma_y(\tau)$ decreases with $\tau^{-1/2}$ showing that it is dominated by a white frequency noise. We obtain a $\sigma_y(\tau)$ equal to 2.6×10^{-9} over a realistic experimental integration time of 30 s. Moreover, one can see that the Flicker noise limit is not reached in our case which show that the measurement could be improved by increasing the integration time.

The reference laser is the heterodyne helium–neon laser described above. To characterize the wavemeter, we have measured the wavelength of a homodyne helium–neon laser system (ML10 GOLD-RENISHAW) (Fig. 3). This measurement serves two purposes: first, the scatter in the data gives an estimate of the statistical error associated with our instrument since both the reference laser and the unknown laser are stabilized to linewidths below 1 MHz (about 3×10^{-9} 1σ), and second, the data tell us if there are any systematic errors associated with our instrument because the difference between the two laser frequencies is already known very precisely [11] . The unknown laser is superimposed to the reference one by autocollimation technique. The frequency of one component of the reference laser has been calibrated in regard to a national reference of the Institut National de Métrologie (LNE-INM-Paris) and is equal to $v_0 = 473.612117\ 6 \times 10^{12}$ Hz. The second component of the beam is separated by 20 MHz. The moving arm of the interferometer has been displaced with a velocity of about 2 mm s^{-1} over a range of 80 mm. Two frequency counters record simultaneously Doppler frequency shifts of both lasers. The reference beam is first filtered with a band-pass filter centered around 20 MHz, demodulated by a clock signal at a frequency of 20 MHz and then sent to the frequency counter. The homodyne laser is only filtered with a low-pass filter (cut-off frequency = 100 kHz). Simultaneously, a weather station measures the room temperature (PT100 thermistor, $\sigma = 5$ mK), the room pressure (Digiquartz, $\sigma = 3$ Pa), humidity content (MH4, General Eastern, $\sigma = 1\%$) and CO2 content (Paroscientifique, $\sigma = 50$ ppm). Using the Edlén equations [12], the refractive index of air is calculated with an uncertainty of 5×10^{-8}. Fig. 4 shows the stability of the Doppler frequency for unknown wavelength laser (for $\tau = 1$ s). Fig. 5 represents the Allan standard deviation calculated using Eq. (3). The slope of the curve is equal to $\tau^{-1/2}$ which characterizes a white noise frequency. We obtain a mean value of $\Delta\nu_U = 6320.57333$ Hz with a relative standard uncertainty 5.6×10^{-8} for $\tau = 30$ s. For the reference laser, we obtain a mean value of $\Delta\nu_R = 6320.56259$ Hz with $\sigma(\tau = 30\ \mathrm{s}) = 2.6 \times 10^{-9}$. As the velocity of the translation stage is locked in onto the frequency of the reference laser, it seems natural to have a better stability for the reference laser than for the unknown laser.

Using Eq. (2), one can deduce the wavelength of the unknown laser. Note that the correction due to the dispersion of air is negligible in this particular case, where the two wavelengths are quite similar. The unknown wavelength has been also measured by beat frequency technique using the national reference and by a commercially available wavemeter (Burleigh WA1000) which has a counting resolution of 0.001 pm over an integration time of 30 s. Table 1 summarizes the wavelength measured by the three methods

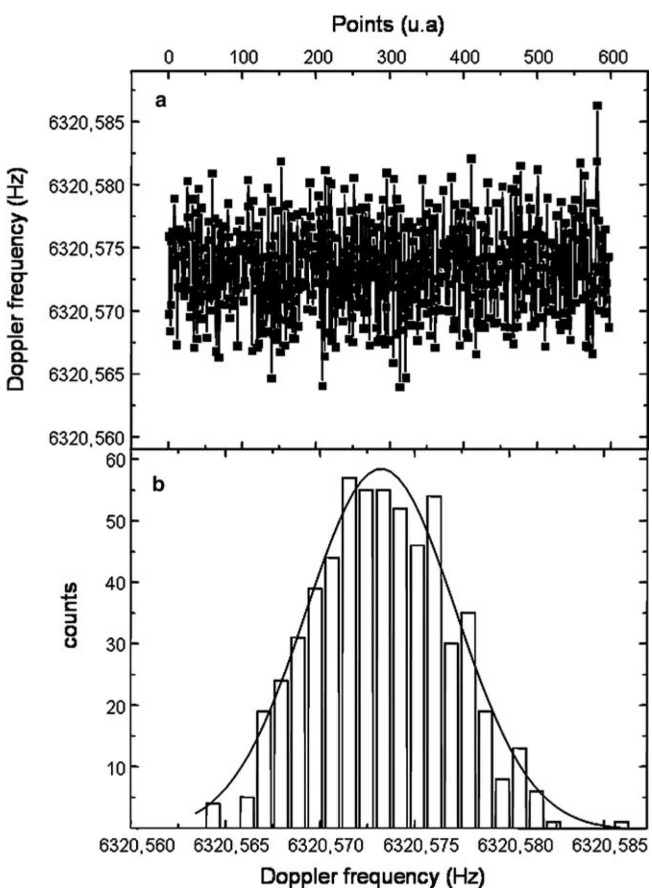

Fig. 4. Statistical error on the Doppler frequency shift of the unknown laser (for $\tau = 1$ s). (a) Set of points with an integration time of 1 s per point. (b) Gaussian distribution of the set of points.

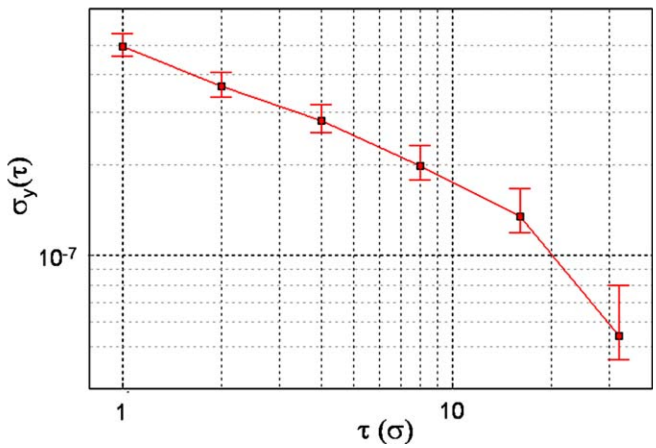

Fig. 5. Allan standard deviation for the Doppler shift of the unknown laser.

and gives also the value of the manufacturer. One can see that the values obtained with our wavemeter, by calibration at the LNE-INM and measured by the manufacturer are in good agreement (within a relative dispersion of 0.2 ppm). On the other hand, the value obtained with the commercial wavemeter is quite different. This is probably due to its limited counting resolution.

3. Systematic and statistical errors

Systematic and statistical errors in interferometry are well known and have been discussed in length by several authors [13,6]. We briefly review the chief source of errors contributing to the total uncertainty. These comments are only intended to be suggestive of the factors entering into the consideration of achievable accuracy; in practice, issues such as beam quality, environmental isolation, materials properties will be of importance as well. Some systematic errors as those caused from plane mirror misalignment (parallel, orthogonal and cosine errors), thermal expansion and flatness of the mirror are negligible if we took into account the ratio in Eq. (2). Parallel error is caused when the mirror is not aligned parallel to the stage travel. Orthogonality error is caused when the mirror axes are not truly orthogonal to each other. Cosine error is caused when the mirror, is not perpendicular to the axis of motion or the laser beam is not parallel to the axis of motion. The total uncertainty includes contributions from the misalignment between the beams, the frequency stability of the lasers, the refractive index of air and the frequency stability of the Doppler shift. These uncertainties σ_i can be added in quadrature to produce an estimate for the uncertainty of the measurement (supposing that each parameters are not correlated):

$$\sigma_\lambda = \sqrt{\sum \sigma_i^2}. \tag{4}$$

The result is summarized in the Table 2. One of the chief cause of systematic error is nonparallelism of the two beams in the interferometer. Any misalignment would

Table 1
Values of the unknown laser wavelength obtained by different methods

	$\Delta\nu$ (nm)	σ_y/y (ppm)	δ (ppm)
This work[a]	632.990581	0.064	0.20
Beating[b]	632.990452	0.0016	0
Wavemeter[c]	632.991	1.6	0.86
Data sheet[d]	632.990577	0.05	0.19

The last column gives the relative discrepancy compared to the value given by the calibration made at the LNE-INM.
[a] Value obtained using the method described in this paper.
[b] Value given by the National Institute of Metrology (Paris).
[c] Value measured using the WA1000 (Burleigh).
[d] Value given by the manufacturer.

Table 2
Systematic and statistical errors contributing the the total uncertainty on the unknown wavelength

Errors	Magnitude or references	In this work
Parallel error	[6,20]	–
Orthogonality error	[6,20]	–
Cosine error	[6,20]	–
Mirror flatness	[6,20]	–
Thermal expansion error	[6]	–
Phase meter linearity	[6]	–
Optical nonlinearity	[6]	–
Wavefront curvature	[6]	–
Contaminants, solvents etc.	[6]	$<10^{-9}$
Short term frequency stability[a]		3×10^{-9}
Accuracy frequency reference[b]		10^{-9}
Electronic drifts	[18,19]	$<10^{-9}$
Index of air	[12]	5×10^{-8}
Doppler shift stability[c]		5.6×10^{-8}
Beams alignment	$\Delta x^2/2L^2$	3.1×10^{-8}
Total		6.4×10^{-8}

[a] Value given by the manufacturer.
[b] Value given by the manufacturer.
[c] Value obtained for $\tau = 30$ s.

cause an increase (or decrease) of the measured ratio leading to an error given by

$$\sigma_{\text{align}} = \frac{1}{2}\left(\frac{\Delta x}{L}\right)^2, \tag{5}$$

where $\Delta x/L$ is the relative angular displacement. We have tried to minimize this error by using the unused output beam of the reference laser (the one on the same side of the beamsplitter in Fig. 3) as a tracer for aligning the unknown laser beam, and checking for parallelism over a distance of about 2 m. We have also found it useful to check for parallelism by looking for a minimum in the measured ratio as the angle of the unknown beam is varied. The optical path length is $L = 2$ m and taking an extreme relative displacement over that distance of $\Delta x = 0.5$ mm, the beam misalignment uncertainty is $\sigma_{\text{align}} = 3.1\times10^{-8}$. A very efficiently autocollimation method based on scanning the retroreflection from the interferometer [15] would permit to decrease this error to the 10^{-8} level. The He–Ne laser wavelength uncertainty is due to the Doppler width of the Ne emission line and to the unknown Ne isotope mixture used in tube. The Doppler width corresponds to a wavelength uncertainty of 2 pm that is $\Delta\lambda_{\text{Doppler}}/\lambda = 3.2\times10^{-6}$. The unknown isotopic mixture gives an uncertainty of approximately 1 pm. Both uncertainties can be minimized by locking the He–Ne laser to an I_2 absorption line [16]. In our case, the larger Doppler uncertainty is more easily reduced by controlling the cavity temperature to obtain equal intensities of two operating longitudinal modes with orthogonal polarization [17]. We also estimate the long term drift of the optoelectronic circuit. The specific system which realizes the phase-shifts have been made with Positive Emitter Coupled Logic technology more suitable for our application because of their long term phase stability. Furthermore, the primary clock of 10 MHz used to generate all the useful signals is an ultra-stable quartz oscillator with a long term stability better than 10^{-9} in relative value for days or even weeks. All the systems has been tested for days and estimated with standard Allan variance [10]. We can assume that long term stability of the optoelectronics is negligible ($\sigma_y(\tau) < 10^{-9}$ for 1 day). Other sources of error like the wavefront curvature in the laser beams and the nonlinearity of the phasemeter which will limit the fringe interpolation, respectively, to 1–4/100 and 1–10/100 in conventional wavemeters [20] are not significant in our case as we measure a ratio of fringe rates rather than a fringe numbers.

Summing the uncertainties in quadrature we see that, in our case, systematic and statistical errors in fringe interpolation limit the accuracy of the wavemeter well above the level allowed by the counting resolution. The error due to beams misalignment is far outweigh all others.

4. Conclusion

We have built a high-accurate wavelength meter based on Doppler effect. The wavelength of a frequency-stabilized helium–neon laser has been measured and the result has been compared to the results obtained with other methods: beat frequency, commercial scanning interferometer-based wavemeter. The counting resolution of our system is equal to 2.6×10^{-9} over an integration time of 30 s without the need of fringe interpolations. Taking into account the statistical errors, the experimental result gives an accuracy for the unknown wavelength of 6.4×10^{-8} (1σ).

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